

SWAMI VIVEKANAND ACADEMY

Class - XII - Maths - Test Paper - Date: 24/12/2019

General Instructions :

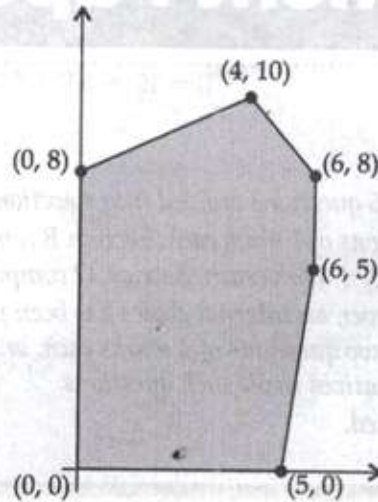
- (i) All the questions are compulsory.
- (ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- (iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

Section 'A'

Q. 1-Q. 10 are multiple choice type questions. Select the correct option.

1. Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is
(a) 1 (b) 2 (c) 3 (d) 4 1
2. The domain of function $\cos^{-1}(2x - 1)$ is
(a) $[0, 1]$ (b) $[-1, 1]$ (c) $(-1, 1)$ (d) $[0, \pi]$ 1
3. The area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units. Then, the value of k will be
(a) 9 (b) 3 (c) -9 (d) 6 1
4. The function $f(x) = \cot x$ is discontinuous on the set
(a) $\{x = n\pi : n \in \mathbb{Z}\}$ (b) $\{x = 2n\pi : n \in \mathbb{Z}\}$ (c) $\left\{x = (2n+1)\frac{\pi}{2} ; n \in \mathbb{Z}\right\}$ (d) $\left\{x = \frac{n\pi}{2} ; n \in \mathbb{Z}\right\}$ 1
5. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is
(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4}$ 1
6. The integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is
(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$ 1

7. The total revenue in ₹ received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$.
The marginal revenue, when $x = 15$ is
(a) 116 (b) 96 (c) 90 (d) 126 1
8. The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3:1 is :
(a) $\frac{3\vec{a} - 2\vec{b}}{2}$ (b) $\frac{7\vec{a} - 8\vec{b}}{4}$ (c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$ 1
9. Let A be a non-singular square matrix of order 3×3 . Then $|\text{adj } A|$ is equal to
(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) $3|A|$ 1
10. The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function.
Minimum of Z occurs at



- (a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10) 1
- (Q. 11-Q. 15) Fill in the blanks.

11. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a 1

OR

- If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to 1

12. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{3}\right)$ is equal to 1

13. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$, is 1

OR

- The slope of tangent to the curve $y = x^2 + 2x + 1$ at $(1, 0)$ is 1

14. The logarithmic function is strictly on $(0, \infty)$. 1

15. The value of $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$ is 1

(Q. 16-Q. 20) Answer the following questions.

16. Integrate : $\int x^2 e^{x^3} dx$ 1

Q17. Find the order and degree of the following differential equation :

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$$

18. State that the given statement is True or False :

Rolle's theorem is applicable for the function $f(x) = |x - 1|$ in $[0, 2]$.

19. Given $P(E) = \frac{1}{2}$ and $P(F) = \frac{1}{5}$, then find $P(E \cup F)$ if E and F are independent events.

20. If $P(A) = P(B)$ and A & B are independent then find the values of $P(A \cap B)$.

OR

State true or false : If A and B are two events such that $P(A) > 0$ and $P(A) + P(B) > 1$, then $P(B/A) \geq 1 - \frac{P(B)}{P(A)}$

Section 'B'

Q21. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then find the value of x .

22. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ and $A = A'$, then find the value of α .

OR

If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c .

23. Integrate the function $\frac{2x}{1+x^2}$.

24. Find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

OR

Solve the differential equation

$$\frac{dy}{dx} + 1 = e^{x+y}$$

25. If E and F are independent events, then show that

(i) E and \bar{F} are independent events.

(ii) \bar{E} and F are also independent events.

26. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Section 'C'

27. Consider $f: R^+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given

by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.

OR

Let $f: N \rightarrow N$ be defined as

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$

for all $n \in N$. State whether the function f is bijective. Justify your answer. 4

28. Differentiate the following function w.r.t. x : 4

$$x^{\sin x} + (\sin x)^{\cos x}$$

29. Find the general solution of the following differential equation : 4

$$y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$$

OR

If $y(x)$ is a solution of the differential equation $\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of 4

$$y\left(\frac{\pi}{2}\right)$$

30. Evaluate $\int \frac{x^2 - 3x + 1}{\sqrt{1 - x^2}} dx$. 4

31. Solve the following linear programming problem graphically :

$$\text{Maximise } Z = 7x + 10y$$

Subject to the constraints

$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$x \geq 10$$

$$x \geq 0, y \geq 0$$

32. Find the variance of the distribution : 4

x	0	1	2	3	4	5
$P(x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

Section 'D'

33. Find the inverse of the following matrix, using elementary transformation 6

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

34. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$. 6

OR

Show that the normal at any point on the curve $x = a \cos \theta + a \theta \sin \theta$, $y = a \sin \theta - a \theta \cos \theta$ is at a constant distance from the origin. 6

35. Find the coordinates of the point P where the line through $A(3, -4, -5)$ and $B(2, -3, 1)$ crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB . 6

OR

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \text{ and the plane } \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5.$$

6

36. Find : $\int \frac{\sqrt{x^2+1}\{\log(x^2+1) - 2\log x\}}{x^4} dx.$

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